

## Measurement and Geometry Strand

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As students relate their experiences from earlier classroom instruction in measurement and geometry to situations in their everyday lives, their knowledge and ability to apply this knowledge increase in depth and sophistication. To demonstrate understanding in this CAHSEE strand, students must be able to select and use appropriate units; estimate and calculate measurements for the length, area, and volume of geometric figures; understand scaling in scale drawings and how changes in linear dimension affect area and volume; and solve problems involving dimensional analysis and conversion from one unit to another.

To demonstrate achievement of the knowledge and skills in the measurement component of this strand, students should be able to use both metric and customary units of measurement for the following:

- determining the relationship between different units within the same system and converting from one unit to another within and between measurement systems
- using scale drawings and models to determine measurements of the original
- solving problems involving dimensional analysis for rates and other compound units
- relating the effect of changing the choice of a linear unit on the related square and cubic units for area and volume, respectively.

The geometry component of this strand includes computing the perimeter, area, and volume of the most common 2- and 3-dimensional figures, and using these common figures to estimate or compute the area of more complex objects.

To demonstrate acquisition of the knowledge and skills in the geometry component of the strand, students should be able to:

- describe, classify, and understand relationships between length, area, and volume among types of 2- and 3-dimensional objects
- use coordinate geometry to represent and examine the properties of figures and their images under translation and reflection
- understand and use the Pythagorean theorem
- recognize and demonstrate understanding of congruence in terms of the sides and angles of 2-dimensional figures.

When CAHSEE items require students to use formulas, the formulas are provided within parentheses in the stem. All formulas the students may use, including the estimated value of  $\pi$  will be provided *except* for the following:

- perimeter of a polygon (sum of the sides)
- circumference of a circle ( $C = 2\pi r$ ,  $C = \pi d$ )
- area of a triangle  $\left(A = \frac{1}{2}bh\right)$
- area of a parallelogram (including rectangles and squares,  $\text{base} \times \text{height}$ )
- volume of a rectangular prism ( $\text{length} \times \text{width} \times \text{height}$ ).

The formula for finding the area of a nontraditional figure such as a rhombus will be provided.

In estimation problems, the approximate value of  $\pi$  that should be used to obtain the correct answer will be provided as part of the stem. Otherwise, the answer will include  $\pi$  (e.g.,  $12 + 3\pi$ ).

The ten California academic content standards covered by the CAHSEE Measurement and Geometry strand are discussed in the following pages.

Strand	<b>Measurement and Geometry (MG)</b>	<b>One millimeter is—</b>
Standard	<b>7MG1.1</b>	<b>A</b> $\frac{1}{1000}$ of a meter.
	<b>Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).</b>	<b>B</b> $\frac{1}{100}$ of a meter.
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	<b>C</b> 100 meters.
		<b>D</b> 1000 meters.
		M00276

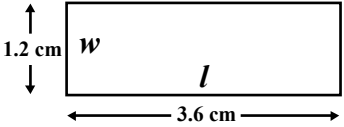
Comparing units and computing the effect of changing units are essential skills for acquiring knowledge in mathematics and science disciplines. CAHSEE test questions for this standard require students to convert between two units of measurement within the same system or between two different systems of measurement. Conversion formulas are provided for test questions that require the student to convert between less commonly used units (such as pints to gallons), systems of measurement, square units, and cubic units.

### *Sample Test Question*

The sample test question requires students to convert from millimeters to meters. The correct answer is choice A. Students must know that to make this conversion, 1 millimeter is equal to  $\frac{1}{1000}$  of a meter.

### *Analysis of Distractors*

The distractors represent error in performing the conversions. Distractor B represents students making the mistake of using  $\frac{1}{100}$  as the conversion factor. Distractor C represents students making the mistake of using 100 as the conversion factor. Distractor D represents students making the mistake of using 1000 as the conversion factor.

Strand	<b>Measurement and Geometry (MG)</b>	<p>The actual width (<math>w</math>) of a rectangle is <b>18 centimeters (cm)</b>. Use the scale drawing of the rectangle to find the actual length (<math>l</math>).</p> 
Standard	<b>7MG1.2</b>	
Constructs	<b>Conceptual Understanding, Problem Solving</b>	

A 6 cm

B 24 cm

C 36 cm

D 54 cm

M02087

The ability to move back and forth between a scale drawing or model and a real object is essential for understanding representation as well as proportional reasoning. Test questions on the CAHSEE focus on the second component of this standard, reading scale drawings and models. Students will be asked to read and interpret drawings and scale models. Students may also be asked to apply given measurements to determine the scale of a figure.

### *Sample Test Question*

The sample test question presents a scale drawing of a rectangle with width 1.2 cm and length 3.6 cm and gives the actual value of the width as 18 cm. The correct answer is choice D.

Students must recognize that the ratio between the width of the scale drawing and the actual width of the rectangle is the same as that between the length of the scale drawing and the actual length. Students may use a variety of approaches to solve the problem, including setting up a proportion similar to  $\frac{1.2}{18} = \frac{3.6}{l}$  and solving for  $l$ .

### *Analysis of Distractors*

The distractors represent errors in using the values presented in the stem. Distractor A results from use of an incorrect relationship between the parts of the proportion:

$(18 \div 3.6) \times 1.2 = 6$ . Distractor B results from subtracting the width from the length and multiplying by 10. Distractor C is  $3.6 \times 10$ , rather than  $3.6 \times 15$ .

Strand	<b>Measurement and Geometry (MG)</b>	<b>Sixty miles per hour is the same rate as which of the following?</b>
Standard	<b>7MG1.3</b> <b>Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.</b>	<b>A</b> 1 mile per minute <b>B</b> 1 mile per second <b>C</b> 6 miles per minute <b>D</b> 360 miles per second
Constructs	<b>Procedural Skills, Conceptual Understanding, Problem Solving</b>	M02473

Dimensional analysis is a critical skill in physical science, engineering, and the social sciences. Students should be familiar with the rates named in the standard as well as other commonly used measures (e.g., kilowatt-hours, foot-pounds, acre-feet). Test questions on the CAHSEE for this standard focus on all three of its components: measures as rates, measures as products, and reasonableness of results.

### *Sample Test Question*

The sample question requires the student to find an equivalent rate to 60 miles per hour. The correct answer is choice A. Students may reason that another way to express the given rate is 60 miles per 60 minutes, since 1 hour is equivalent to 60 minutes. The expression is 60 miles/hour is equivalent to 60 miles/60 minutes and that since  $60 \div 60 = 1$ , the rate is equivalent to 1 mile/minute. Students should also be encouraged to evaluate the answer choices in the question for their reasonableness as a restatement of this real-world rate.

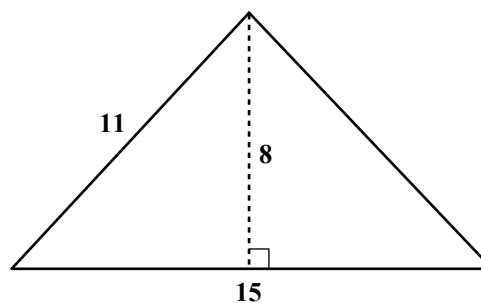
### *Analysis of Distractors*

The distractors represent misunderstandings of the required calculations. Distractor B results from incorrectly equating 1 hour to 60 seconds. Distractor C is obtained by dividing 60 miles per hour by 10, rather than by 60. Distractor D is obtained by multiplying 60 by 6.

Strand **Measurement and Geometry (MG)**

Standard **7MG2.1**  
**Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.**

Constructs **Procedural Skills, Conceptual Understanding**



**What is the area of the triangle shown above?**

- A 44 square units
- B 60 square units
- C 88 square units
- D 120 square units

M00101

All students should know how to compute the area and volume of basic figures and also how to apply basic formulas in many areas to solve problems. This standard requires students to find the perimeter and area of 2-dimensional figures and the surface area and volume of 3-dimensional figures. The figures tested on the CAHSEE include parallelograms, trapezoids, triangles, circles, prisms, and cylinders. Students are required to know the following formulas:

- perimeter of a polygon (sum of the sides)
- circumference of a circle ( $C = 2\pi r$ ,  $C = \pi d$ )
- area of a triangle  $\left(A = \frac{1}{2}bh\right)$
- area of a parallelogram (including rectangles and squares,  $\text{base} \times \text{height}$ )
- volume of a rectangular prism ( $\text{length} \times \text{width} \times \text{height}$ ).

These formulas will not be given with the test questions. The estimated value of  $\pi$  will be given unless  $\pi$  is included in the answer choices.

### *Sample Test Question*

The sample question presents a triangle with the dimensions labeled and asks students to compute the area. The correct answer is choice B. Students should know the area

formula  $A = \frac{1}{2}bh$  and use it to determine that the height is 8 units and the base is 15 units. The area is 60 square units.

*Analysis of Distractors*

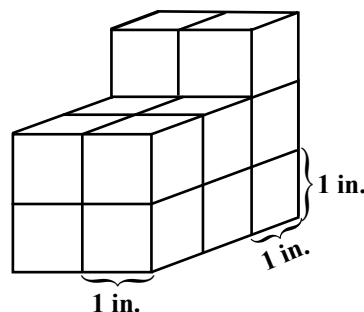
The distractors represent misapplications of the area formula. Distractor A uses 11 as the base, rather than 15. Distractor C uses 11 as the base and also fails to take  $\frac{1}{2}$  of the product of the base and height. Distractor D applies the formula to the correct dimensions but contains the error of failing to take  $\frac{1}{2}$  of the product of the base and height.

Strand            **Measurement and  
Geometry (MG)**

Standard        **7MG2.2**  
**Estimate and compute the**  
**area of more complex or**  
**irregular two-and three-dimensional**  
**figures by breaking the figures down**  
**into more basic geometric objects.**

Constructs       **Procedural Skills,**  
**Conceptual**  
**Understanding**

**One-inch cubes are stacked as shown in the drawing below.**



**What is the total surface area?**

- A**   19 in.<sup>2</sup>
- B**   29 in.<sup>2</sup>
- C**   32 in.<sup>2</sup>
- D**   38 in.<sup>2</sup>

M02812

This standard emphasizes the development of problem-solving skills with visual tools. CAHSEE test questions for this standard focus on both of its components. The first component requires students to identify the measurable and/or computable parts of a shape or structure. This identification task is largely dependent on the ability to visualize the familiar geometric structures that make up a more complex figure. Where estimation is required, students may be given dimensions of known shapes, scale and proportion, or grids in the visual prompt. The second component requires students to estimate and/or compute the area of the subdivided portions of a figure. Frequently, the dimensions of the component parts are not given directly by labeled measurements but must be determined by such means as adding or subtracting lengths or extending lines.

### *Sample Test Question*

The sample question presents a drawing of an assembly of 1-inch cubes and asks students to determine the surface area. The correct answer is choice D. The foreground portion of the object contains a 2 by 2 by 2-cube grouping, and the background contains a 1 by 2 by 3-cube grouping. To find the surface area, students must visually assemble the surface of the individual cubes that make up the entire object and recognize that the object has 8 planar faces: right, rear, left, front, bottom, front top, rear face, and rear top. The surface areas of each corresponding face are 7, 6, 7, 4, 6, 4, 2, and 2 square inches, and the sum of the areas of these faces is 38 square inches.



*Analysis of Distractors*

The distractors represent inaccurate visualizations of the faces of the object. Distractor A includes only half of the exterior faces. Distractor B fails to include either the left or right face and either the rear face or rear top. Distractor C fails to include either the bottom or rear of the figure.

Strand

**Measurement and  
Geometry (MG)**

Standard

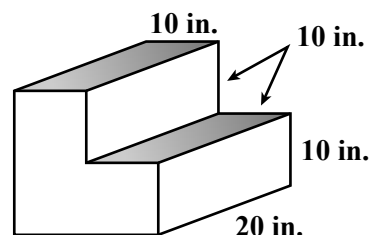
**7MG2.3**

**Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and volume is multiplied by the cube of the scale factor.**

Constructs

**Procedural Skills,  
Conceptual  
Understanding,  
Problem Solving**

The short stairway shown below is made of solid concrete. The height and width of each step is 10 inches (in.). The length is 20 inches.



What is the volume, in cubic inches, of the concrete used to create this stairway?

- A 3000
- B 4000
- C 6000
- D 8000

M02990

This standard contains two components. The first component focuses on the students' ability to compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids.

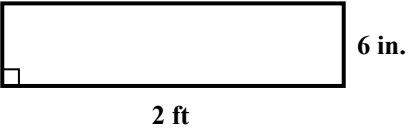
The second component focuses on the students' understanding of the results of multiplying the lengths of all dimensions by a scale factor, i.e., the surface area is multiplied by the square of the scale factor or the volume is multiplied by the cube of the scale factor. Items addressing the scale factor component of this standard may include two-dimensional objects.

### *Sample Test Question*

The sample question shows a stairway made from 2 rectangular prisms and asks students to find the volume of the concrete used to build the stairway. The correct answer is choice C. Students should find the volume of the larger rectangular prism by multiplying 10 times 20 times 20. They should then find the volume of the smaller rectangular prism by multiplying 10 times 10 times 20. The sum of these 2 volumes gives the correct answer.

### *Analysis of Distractors*

The distractors offer solutions that use the values of the problem incorrectly and thus reflect a misunderstanding of the concept. Distractor A represents the error of finding the volume of the larger prism by multiplying 10 times 10 times 20 and the volume of the smaller prism by multiplying 10 times 10 times 10. Distractor B represents the error of finding only the volume of the larger prism. Distractor D represents the error of doubling the volume of the larger prism.

Strand	<b>Measurement and Geometry (MG)</b>	<p>The width of the rectangle shown below is <b>6 <u>inches</u> (in.)</b>. The length is <b>2 <u>feet</u> (ft)</b>.</p>  <p>What is the area of the rectangle in square <b><u>inches</u></b>?</p> <p>A 12 B 16 C 60 D 144</p> <p>M03243</p>
Standard	<b>7MG2.4</b>	
	<b>Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or <math>[1 \text{ ft}^2] = [144 \text{ in}^2]</math>, 1 cubic inch is approximately 16.38 cubic centimeters or <math>[1 \text{ in}^3] = [16.38 \text{ cm}^3]</math>).</b>	
Construct	<b>Conceptual Understanding</b>	

Unit conversions for area and volume have practical applications in students' lives. CAHSEE test questions for this standard require students to relate the changes in measurement with a change of scale to the units used and to convert between units. The emphasis is on the relationship between linear units for distance, square units for area, and cubic units for volume. Conversion formulas will be provided for linear measures except for inches to feet to yards or conversions within the metric system.

### *Sample Test Question*

The sample question gives a diagram of a rectangle in inches and feet and asks students for the area in square inches. The correct answer choice is D. Students should convert 2 feet to 24 inches and multiply 6 and 24 to get the area of the rectangle.

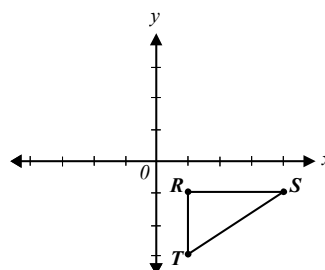
### *Analysis of Distractors*

The distractors represent incorrect use of the values given in the problem. Distractor A represents the error of multiplying 6 and 2. Distractor B represents the error of finding the perimeter of the rectangle using the given values without making a conversion of feet to inches. Distractor C represents the error of making the conversion of feet to inches, but then finding the perimeter of the rectangle.

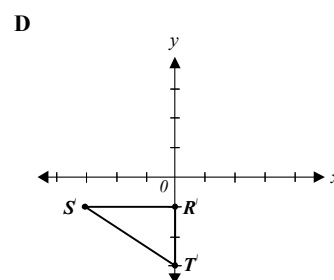
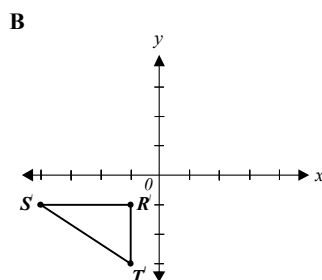
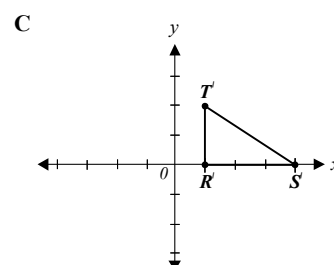
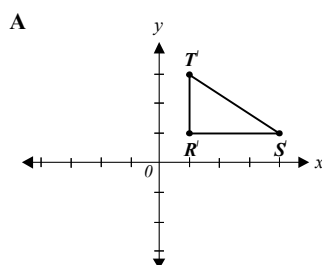
Strand **Measurement and Geometry (MG)**

Standard **7MG3.2**  
**Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



Which of the following triangles  $R'S'T'$  is the image of triangle  $RST$  that results from reflecting triangle  $RST$  across the  $y$ -axis?



M02861

Understanding the  $xy$  coordinate system is an essential step in developing important thinking and problem-solving skills. Visualizing and manipulating objects are useful tools, not only in geometry, but also in other disciplines.

CAHSEE test questions for this standard assess students' understanding of all components of the standard: plotting with ordered pairs, determining lengths and areas from plotted figures, and finding images following transformations by translations and reflections.

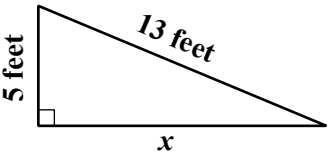
### *Sample Test Question*

The sample question requires students to choose the correct transformation of an object (triangle  $RST$ ) by reflecting it across the  $y$ -axis. The correct answer is choice B. Students must recognize the reflective correspondence between points  $R$  and  $R'$ ,  $S$  and  $S'$ , and  $T$  and  $T'$  and that triangle  $R'S'T'$  is the reflective image, across the  $y$ -axis, of triangle  $RST$ .

*Analysis of Distractors*

Distractor A is the reflection of triangle  $RST$  across the  $x$ -axis. Distractor C is the reflection of triangle  $RST$  across the  $x$ -axis and its translation by one unit down, or its reflection

across  $y = -\frac{1}{2}$ . Distractor D is the reflection of triangle  $RST$  across the  $y$ -axis and its translation by 1 unit to the right.

Strand	<b>Measurement and Geometry (MG)</b>	<b>What is the value of <math>x</math> in the right triangle shown below?</b>
Standard	<b>7MG3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.</b>	<div data-bbox="948 333 1273 485">  </div> <div data-bbox="786 522 915 701"> <p>A 8 feet</p> <p>B 12 feet</p> <p>C 18 feet</p> <p>D 23 feet</p> </div>
Constructs	<b>Conceptual Understanding, Problem Solving</b>	M03181

The Pythagorean theorem is important for its problem-solving function as well as its role as a bridge between geometry and algebra. CAHSEE test questions assessing this standard address the following components of the standard: using the Pythagorean theorem to find the length of the missing base, altitude, or hypotenuse of a right triangle and using the Pythagorean theorem to find lengths of line segments in figures other than triangles.

### *Sample Test Question*

The sample test question gives a right triangle with a hypotenuse of 13 feet, an altitude of 5 feet, and an unknown base. Students should use the Pythagorean Theorem  $5^2 + x^2 = 13^2$  to obtain the correct answer of 12 which is choice B.

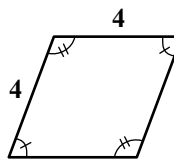
### *Analysis of Distractors*

Distractor A is the difference of the hypotenuse and altitude. Distractor C is the sum of the hypotenuse and altitude. Distractor D is the sum of the hypotenuse and twice the altitude.

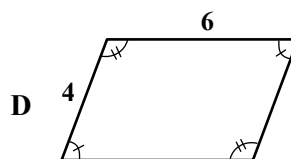
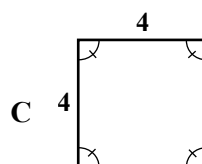
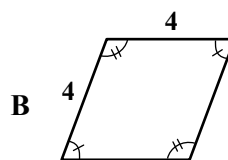
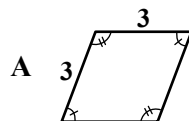
Strand **Measurement and Geometry (MG)**

Standard **7MG3.4**  
**Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.**

Construct **Conceptual Understanding**



Which figure is congruent to the figure shown above?



M00020

CAHSEE test questions for this standard require students to discriminate between figures that are or are not congruent to a given figure. Students may also be asked to identify specific reasons to support the assertion that two figures are congruent. Relationships between sides and angles of figures are significant because they may indicate the presence or absence of congruence by axiomatic reasoning, such as side-angle-side congruence. Items in this standard will not involve the use of acronyms for side and angle relationships of congruent figures (e.g. SAS, SSS).

### *Sample Test Question*

The sample question presents a figure with the dimensions of the sides given and the angles marked. Students are asked to use the characteristics of this figure to find the congruent figure among the answer choices. The correct answer is choice B. Students should understand that two

polygons are congruent if all corresponding sides and angles are congruent. Reasoning from this basis, students should recognize that the figure in choice B is congruent to the given figure because of the correspondence with sides of length 4 and angles with two marks. Students should also understand that because both figures are rhombi with non-congruent adjacent angles, all four sides are congruent and the opposite angles are congruent.

*Analysis of Distractors*

Distractor A provides a similar, but not congruent, figure. Distractor C is a square, and Distractor D is a parallelogram with non-congruent adjacent sides.